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# Magnetic moments of baryons in a quark model

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**Abstract.** Magnetic moments of octet baryons are calculated in a non-relativistic quark model with scalar binding, using a model for SU(3) breaking developed earlier. Violation of ordinary spin symmetry is also considered, leading to a mixture of spin–orbital states. Consistency relations are derivable. In a special case, restricted to the **56**, the results are given explicitly.

## 1. Introduction

A single-particle model has been discussed elsewhere (James 1968, to be referred to as I), in which the quarks are bound by a scalar potential $-U$, regarded for simplicity as a very deep square well. The change in energy for a $\lambda$ quark

$$\delta E = (M-U)\frac{\delta M}{E_0}+\frac{p^2}{2E_0{}^3}(\delta M)^2+\ldots$$

depends not only on the change in mass but also on the potential. This can produce a $\Lambda$–$\Sigma$ splitting in the baryon octet, because the effective potential seen by the $\lambda$ quark is different in the two cases. The SU(6) mass formula can be reproduced, and mass relations can be found for resonances of higher spin which agree with available data.

In this model the potentials need violate only the ordinary spin part of SU(6); it is no longer necessary to include large SU(3)-breaking interactions. The masses of the baryon octet can be expressed in terms of two parameters, $E_1$ and $E_2$, together with energy increments $\delta_1$ and $\delta_2$, referring to the spin triplet and singlet states respectively. These parameters can be estimated as

$$\begin{aligned} E_1 &= 412\,\text{MeV}, & E_2 &= 216\,\text{MeV} \\ \delta_1 &= 135\,\text{MeV}, & \delta_2 &= 280\,\text{MeV}. \end{aligned} \tag{1}$$

It is well known that the magnetic moments of quarks may be enhanced by strong binding in a scalar field (Bogolubov, Struminski and Tavkhelidze 1965, unpublished, Lipkin and Tavkhelidze 1965). If the effective potential is assumed to be entirely scalar (Tavkhelidze 1965), the magnetic moments of nucleons calculated in a non-relativistic model agree closely with experiment. From our values (1), following the procedure of Morpurgo (1965), we readily find

$$\mu(\mathrm{P}) = 2{\cdot}86\text{ n.m.,} \qquad \mu(\mathrm{N}) = -2{\cdot}02\text{ n.m.}$$

It is suggested in I that the potential may contain a small vector component, which cannot be neglected in calculating the moments. We should also consider the possibility of spin-orbital states other than the $^2$S state.

The observed enhancement of the $g$ factor may, of course, be due to a charged meson cloud surrounding the quark (cf., for example, Morpurgo 1966). In the present paper, however, we consider only the simplest possibility, treating the quarks as independent Dirac particles with $g$ factors enhanced by scalar binding. We include SU(3) violation and also violation of ordinary spin symmetry, giving rise to a mixture of spin–orbital states. This simple model is amenable to calculation; for the baryon octet, it leads to consistency relations which should provide a test for the model.

## 2. Single-particle model

Our assumptions will be as follows:

(i) The quarks within a hadron behave as free Dirac particles, except for $g$-factor enhancement caused by scalar binding.

(ii) The contribution of exchange currents and meson clouds can be neglected.

(iii) The spin–orbital states are described by the non-relativistic SU(6) classification (cf. Dalitz 1965, Dalitz 1966, unpublished).

(iv) The relativistic correction to the magnetic moments can be neglected.

We may then obtain magnetic moments for the quarks, assuming that SU(3) breaking is caused only by the $\lambda$-quark mass increment (see discussion in I). The magnetic moments are

$$\boldsymbol{\mu}_i = g_i(\boldsymbol{L}_i + 2\boldsymbol{S}_i)$$

where

$$g_{\mathrm{p}} = \frac{e}{3\epsilon}, \qquad g_{\mathrm{n}} = \frac{-e}{6\epsilon}, \qquad g_\lambda = \frac{-e}{6(\epsilon+\delta)} \tag{2}$$

$$(\hbar = c = 1).$$

The parameter $\epsilon$ is related to the energy of the quark but is not the same unless the vector potential vanishes. It is shown in I that the appropriate combinations for the baryon octet are

$$\begin{aligned}
&\mathrm{P}\begin{cases} \mathrm{p}: & \tfrac{1}{8}\epsilon_{\mathrm{b}} \\ \mathrm{n}: & \tfrac{1}{4}\epsilon_{\mathrm{a}} \end{cases} \\
&\mathrm{N}\begin{cases} \mathrm{p}: & \tfrac{1}{4}\epsilon_{\mathrm{a}} \\ \mathrm{n}: & \tfrac{1}{8}\epsilon_{\mathrm{b}} \end{cases} \\
&\Lambda\begin{cases} \mathrm{p,\,n}: & \tfrac{1}{8}(3\epsilon_1+5\epsilon_2) \\ \lambda: & \tfrac{1}{4}(3\epsilon_1+\epsilon_2)+\tfrac{1}{4}(3\delta_1+\delta_2) \end{cases} \\
&\Sigma\begin{cases} \mathrm{p,\,n}: & \tfrac{1}{8}\epsilon_{\mathrm{b}} \\ \lambda: & \tfrac{1}{4}(\epsilon_{\mathrm{a}}+\delta_{\mathrm{a}}) \end{cases} \\
&\Xi\begin{cases} \mathrm{p,\,n}: & \tfrac{1}{4}\epsilon_{\mathrm{a}} \\ \lambda: & \tfrac{1}{8}(\epsilon_{\mathrm{b}}+\delta_{\mathrm{b}}) \end{cases}
\end{aligned} \tag{3}$$

where

$$\epsilon_{\mathrm{a}} = \epsilon_1+3\epsilon_2, \qquad \epsilon_{\mathrm{b}} = 5\epsilon_1+3\epsilon_2$$
$$\delta_{\mathrm{a}} = \delta_1+3\delta_2, \qquad \delta_{\mathrm{b}} = 5\delta_1+3\delta_2.$$

The values of $\delta_1$ and $\delta_2$ (energy increments attached solely to the $\lambda$ quark) are given in equations (1).

## 3. Structure of the baryon octet

The structure of three-body nuclei and the magnetic moments of the charge doublet states $^3$H and $^3$He have been discussed extensively (Sachs and Schwinger 1946, Sachs 1947, Rosenfeld 1948, Sachs 1953, Derrick and Blatt 1958). With respect to permutations, the spatial function may be symmetrical (S), antisymmetrical (A) or of mixed symmetry ($\mathrm{M}^+$, $\mathrm{M}^-$). The last functions satisfy permutational symmetry in two coordinates:

$$P_{23}\bar{X} = \bar{X} \qquad (\mathrm{M}^+)$$
$$P_{23}X = -X \quad (\mathrm{M}^-).$$

For the spatial dependence we have a complete set of orthogonal functions ($X_{\mathrm{s}}$, $X_{\mathrm{a}}$, $\bar{X}$, $X$), belonging to these symmetry classes. Unless we make the strong assumption of para-statistics (Greenberg 1964), we must, of course, assign the baryon **56** to an antisymmetric spatial state, which is expected to be mainly an S state.

The spin functions can be classified in a similar way (see, for example, Sachs 1953):

$$\begin{aligned} \phi_s &= \alpha_1\alpha_2\beta_3+\alpha_1\beta_2\alpha_3+\beta_1\alpha_2\alpha_3, & S &= \tfrac{3}{2} \\ \bar{\phi} &= -2(\alpha_1\beta_2\alpha_3-2\beta_1\alpha_2\alpha_3+\alpha_1\alpha_2\beta_3), & S &= \tfrac{1}{2} \\ \phi &= \alpha_1(\alpha_2\beta_3-\alpha_3\beta_2), & S &= \tfrac{1}{2}. \end{aligned} \tag{4}$$

The unitary spin functions for the baryon octet belong to the mixed symmetry representation and will be denoted by $\bar{\eta}$ and $\eta$:

$$\begin{aligned} &\mathrm{N}\begin{cases} \bar{\eta} = -2(n_1n_2p_3-2p_1n_2n_3+n_1p_2n_3) \\ \eta = n_1(n_2p_3-n_3p_2) \end{cases} \\ &\Lambda\begin{cases} \bar{\eta} = 6\{\lambda_3(p_1n_2-p_2n_1)-\lambda_2(p_3n_1-p_1n_3)\} \\ \eta = \lambda_3(p_1n_2-p_2n_1)+\lambda_2(p_3n_1-p_1n_3)-2\lambda_1(p_2n_3-p_3n_2) \end{cases} \\ &\Sigma^0\begin{cases} \bar{\eta} = -2\{\lambda_3(p_1n_2+p_2n_1)+\lambda_2(p_3n_1+p_1n_3)-2\lambda_1(p_2n_3+p_3n_2)\} \\ \eta = \lambda_3(p_1n_2+p_2n_1)-\lambda_2(p_3n_1+p_1n_3) \end{cases} \\ &\Xi^0\begin{cases} \bar{\eta} = -2(\lambda_1\lambda_2p_3-2p_1\lambda_2\lambda_3+\lambda_1p_2\lambda_3) \\ \eta = \lambda_1(\lambda_2p_3-\lambda_3p_2). \end{cases} \end{aligned} \tag{5}$$

The normalization is fixed by the relations (cf. Sachs 1953)

$$\begin{aligned} P_{12}\eta &= P_{13}\eta = \tfrac{1}{2}\eta+\tfrac{1}{4}\bar{\eta} \\ P_{12}\bar{\eta} &= P_{13}\bar{\eta} = 3\eta-\tfrac{1}{2}\bar{\eta}. \end{aligned}$$

To construct wave functions for the octet, we therefore have the following sets:

$$\begin{gathered} X_s, \quad X_a, \quad \bar{X}, \quad X \\ \phi_s, \quad \bar{\phi}, \quad \phi \\ \bar{\eta}, \quad \eta. \end{gathered} \tag{6}$$

We also require that the total angular momentum add up to $J=\frac{1}{2}$. For $L=0$ we are restricted to $S=\frac{1}{2}$, i.e. the spin functions $\bar{\phi}$ and $\phi$. For $L=2$ we can have only $S=\frac{3}{2}$, i.e. the function $\phi_s$, while for $L=1$ there is no restriction.

By combining functions from the three symmetry classes, we may construct further functions with definite symmetry (Sachs 1953). Considering the two sets ($F_s$, $F_a$, $\bar{F}$, $F$) and ($G_s$, $G_a$, $\bar{G}$, $G$), we can form the symmetrical products

$$F_sG_s, \quad F_aG_a, \quad 12FG+\bar{F}\bar{G}.$$

The antisymmetrical products are

$$F_sG_a, \quad F_aG_s, \quad F\bar{G}-\bar{F}G.$$

Finally, the products of intermediate symmetry are

$$F\bar{G}+\bar{F}G, \quad 12FG-\bar{F}\bar{G}$$

as can be seen by applying $P_{12}$ and $P_{13}$ to the factors.

From expressions (6) we can therefore construct the following antisymmetrized wave functions:

$$\begin{aligned} &X_a(12\phi\eta+\bar{\phi}\bar{\eta}): && {}^2\mathrm{S}(56), \quad {}^2\mathrm{P}(56) \\ &X_s(\phi\bar{\eta}-\bar{\phi}\eta): && {}^2\mathrm{S}(20), \quad {}^2\mathrm{P}(20) \\ &X(12\phi\eta-\bar{\phi}\bar{\eta})-\bar{X}(\phi\bar{\eta}+\bar{\phi}\eta): && {}^2\mathrm{S}(70), \quad {}^2\mathrm{P}(70) \\ &(X\bar{\eta}-\bar{X}\eta)\phi_s: && {}^4\mathrm{P}(70), \quad {}^4\mathrm{D}(70). \end{aligned}$$

The SU(6) classification is indicated according to spin/unitary spin symmetry:

$$\mathrm{S}(56), \quad \mathrm{A}(20), \quad \mathrm{M}(70).$$

We may choose particle 1 uniquely according to the scheme

$$\bar{\eta}(\mathrm{N}) = p_1 n_2 n_3, \qquad \bar{\eta}(\Sigma^0) = \lambda_1(p_2 n_3 + p_3 n_2), \qquad \bar{\eta}(\Xi^0) = p_1 \lambda_2 \lambda_3, \qquad \eta(\Lambda) = \lambda_1(p_2 n_3 - p_3 n_2)$$

with corresponding choices for the other charge states. From equations (5) it is seen that the unitary spin function $\eta$ vanishes for all states except the $\Lambda$ hyperon; in this case $\bar{\eta}$ vanishes instead. The spin–orbital functions then reduce to those of table 1. The wave

**Table 1. Spin–orbital functions for the baryon octet**

| | $^2$S, $^2$P(56) | $^2$S, $^2$P(20) | $^2$S, $^2$P(70) | $^4$P, $^4$D(70) |
|---|---|---|---|---|
| N, $\Sigma$, $\Xi$ | $X_a\bar{\phi}$ | $X_s\phi$ | $X\bar{\phi}+\bar{X}\phi$ | $X\phi_s$ |
| $\Lambda$ | $X_a\phi$ | $X_s\bar{\phi}$ | $12X\phi-\bar{X}\bar{\phi}$ | $\bar{X}\phi_s$ |

function of the baryon octet will be a linear combination of these functions, the actual mixture depending on the type of non-central force (Rosenfeld 1948). We take the combination

$$\psi = \sum_{i=1}^{8} a_i^{1/2} \psi_i$$

as set out in table 2.

**Table 2. Matrix elements for N, $\Sigma$ and $\Xi$**

| State | $^2$S(56) | $^2$S(20) | $^2$S(70) | $^2$P(56) | $^2$P(20) | $^2$P(70) | $^4$P(70) | $^4$D(70) |
|---|---|---|---|---|---|---|---|---|
| Weight | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ | $a_8$ |
| $\langle L_1{}^z \rangle$ | 0 | 0 | 0 | $\frac{2}{9}$ | $\frac{2}{9}$ | $\frac{2}{9}$ | $-\frac{1}{9}$ | $\frac{1}{3}$ |
| $\langle S_1{}^z \rangle$ | $-\frac{1}{6}$ | $\frac{1}{2}$ | $\frac{1}{6}$ | $\frac{1}{18}$ | $-\frac{1}{6}$ | $-\frac{1}{18}$ | $\frac{5}{18}$ | $-\frac{1}{6}$ |
| $\langle L_2{}^z \rangle$ | 0 | 0 | 0 | $\frac{2}{9}$ | $\frac{2}{9}$ | $\frac{2}{9}$ | $-\frac{1}{9}$ | $\frac{1}{3}$ |
| $\langle S_2{}^z \rangle$ | $\frac{1}{3}$ | 0 | $\frac{1}{6}$ | $-\frac{1}{9}$ | 0 | $-\frac{1}{18}$ | $\frac{5}{18}$ | $-\frac{1}{6}$ |

In the case of $\Lambda$ the values for the **20** and **56** representations should be interchanged.

A complication arises for the $\Lambda$ hyperon, as can be seen by inspection of table 1; the functions for the $\Lambda$ have a special form, differing from the other members of the octet. For the $^2\{8\}$ reduction of the **70**, in particular, they have an apparently different weighting with respect to $\bar{X}$, $X$ and $\bar{\phi}$, $\phi$. We must therefore decide on the relative weight of these functions. By analogy with the mixed symmetry functions (4), this is given by

$$\frac{(\bar{X}, \bar{X})}{(X, X)} = \frac{(\bar{\phi}, \bar{\phi})}{(\phi, \phi)} = 12.$$

It can then be seen that, for the states $^2$S(70) and $^2$P(70), matrix elements of orbital and spin operators are to be averaged equally over both types of mixed symmetry function.

## 4. Magnetic moments of the baryon octet

We wish to evaluate matrix elements for the states of table 2, in analogy with the procedure for nuclei (Sachs and Schwinger 1946, Sachs 1947). The diagonal elements of the spin operators, $\langle S_1{}^z \rangle$ and $\langle S_2{}^z \rangle = \langle S_3{}^z \rangle$, may be evaluated explicitly from equations (4). For the orbital angular momentum, however, further results are needed.

The position vectors ($\boldsymbol{r}$, $\boldsymbol{\rho}$), with

$$\boldsymbol{r} = \frac{1}{\sqrt{6}}(\boldsymbol{r}_2+\boldsymbol{r}_3-2\boldsymbol{r}_1)$$

$$\boldsymbol{\rho} = \frac{1}{\sqrt{2}}(\boldsymbol{r}_2-\boldsymbol{r}_3)$$

form the basis for an M representation of this permutation group. The functions $\bar{X}$ and $X$ are then either symmetrical or antisymmetrical under interchange of $\boldsymbol{r}$ and $\boldsymbol{\rho}$. It was observed by Sachs and Schwinger that use can be made of this property, since the orbital operators are given by

$$L^z = \frac{1}{\mathrm{i}}\left\{\left(x\frac{\partial}{\partial y}-y\frac{\partial}{\partial x}\right)+\left(u\frac{\partial}{\partial v}-v\frac{\partial}{\partial u}\right)\right\} \tag{7}$$

$$L_1{}^z = \frac{2}{3\mathrm{i}}\left(x\frac{\partial}{\partial y}-y\frac{\partial}{\partial x}\right)$$

where

$$\boldsymbol{r} = (x, y, z), \qquad \boldsymbol{\rho} = (u, v, w).$$

The two terms in equation (7) clearly have the same diagonal matrix element for a state with definite symmetry under interchange of $\boldsymbol{r}$ and $\boldsymbol{\rho}$. Combining these two equations, we obtain

$$\langle L_1{}^z\rangle = \tfrac{1}{3}\langle L^z\rangle.$$

This result is obtained more simply for the other spatial functions, which are either symmetrical or antisymmetrical in all three coordinates. It follows that

$$\langle L_1{}^z\rangle = \langle L_2{}^z\rangle = \langle L_3{}^z\rangle.$$

The total orbital angular momentum may be evaluated from the usual vector addition formula (see, for example, Preston 1962):

$$\langle L^z\rangle = \frac{\langle J^z\rangle}{2J(J+1)}\{J(J+1)+L(L+1)-S(S+1)\}.$$

Thus we can evaluate the diagonal elements of interest: the results are given in table 2.

For the baryons N, $\Sigma$ and $\Xi$, we obtain

$$\begin{aligned} F_1 &= \langle L_1{}^z+2S_1{}^z\rangle \\ &= \tfrac{1}{9}(-3a_1+9a_2+3a_3+3a_4-a_5+a_6+4a_7) \\ F_2 &= \langle L_2{}^z+2S_2{}^z\rangle = \langle L_3{}^z+2S_3{}^z\rangle \\ &= \tfrac{1}{9}(6a_1+3a_3+2a_5+a_6+4a_7). \end{aligned} \tag{8}$$

For the $\Lambda$ hyperon

$$\begin{aligned} G_1 &= \langle L_1{}^z+2S_1{}^z\rangle \\ &= \tfrac{1}{9}(9a_1-3a_2+3a_3-a_4+3a_5+a_6+4a_7) \\ G_2 &= \langle L_2{}^z+2S_2{}^z\rangle = \langle L_3{}^z+2S_3{}^z\rangle \\ &= \tfrac{1}{9}(6a_2+3a_3+2a_4+a_6+4a_7). \end{aligned} \tag{9}$$

These results give only the diagonal elements. We must also consider cross terms of the form

$$\langle i|L_1{}^z+2S_1{}^z|j\rangle$$

and

$$\langle i|L_2{}^z+2S_2{}^z|j\rangle = \langle i|L_3{}^z+2S_3{}^z|j\rangle$$

where $i \neq j$. Without a detailed knowledge of the spatial functions we cannot evaluate these elements (see discussion by Sachs (1947) for the case of $^3$H). We can take them into account, however, at the expense of introducing an extra parameter. In each case the matrix element of $L^z$ vanishes:

$$\langle i|L_1{}^z + L_2{}^z + L_3{}^z|j\rangle = 0$$

since $\psi_i$ and $\psi_j$ are orthogonal eigenfunctions of this operator. The same applies to total spin:

$$\langle i|S_1{}^z + S_2{}^z + S_3{}^z|j\rangle = 0.$$

We therefore introduce a parameter

$$\begin{aligned}\alpha &= \sum_{i\neq j} \langle i|L_2{}^z + 2S_2{}^z) + (L_3{}^z + 2S_3{}^z)|j\rangle \\ &= -\sum_{i\neq j} \langle i|L_1{}^z + 2S_1{}^z|j\rangle \qquad (10)\end{aligned}$$

in terms of which the non-diagonal elements can be expressed. Because of the special form of its wave function, the $\Lambda$ hyperon has in general a different parameter, which we call $\beta$.

From equations (2), (3), (8), (9) and (10) we can now calculate the magnetic moments of the baryon octet:

$$\mu(\mathrm{P}) = -\tfrac{2}{3}e\left(\frac{a}{\epsilon_a} - 4\frac{b}{\epsilon_b}\right), \qquad \mu(\mathrm{N}) = \tfrac{4}{3}e\left(\frac{a}{\epsilon_a} - \frac{b}{\epsilon_b}\right)$$

$$\mu(\Lambda) = -\tfrac{2}{3}e\left(\frac{G_1 - \beta}{3\epsilon_1 + \epsilon_2 + 685} - \frac{2G_2 + \beta}{3\epsilon_1 + 5\epsilon_2}\right)$$

$$\mu(\Sigma^0) = -\tfrac{2}{3}e\left(\frac{a}{\epsilon_a + 980} - \frac{b}{\epsilon_b}\right) \qquad (11)$$

$$\mu(\Sigma^+) = -\tfrac{2}{3}e\left(\frac{a}{\epsilon_a + 980} - 4\frac{b}{\epsilon_b}\right), \qquad \mu(\Sigma^-) = -\tfrac{2}{3}e\left(\frac{a}{\epsilon_a + 980} + 2\frac{b}{\epsilon_b}\right)$$

$$\mu(\Xi^0) = \tfrac{4}{3}e\left(\frac{a}{\epsilon_a} - \frac{b}{\epsilon_b + 1520}\right), \qquad \mu(\Xi^-) = -\tfrac{2}{3}e\left(\frac{a}{\epsilon_a} + 2\frac{b}{\epsilon_b + 1520}\right)$$

where

$$a = F_1 - \alpha, \qquad b = 2F_2 + \alpha$$

$$\epsilon_a = \epsilon_1 + 3\epsilon_2, \qquad \epsilon_b = 5\epsilon_1 + 3\epsilon_2.$$

In these units $e$ takes the value $2 \times 939$ Mev (nuclear magneton).

## 5. Discussion and conclusion

Excluding the equation for $\mu(\Lambda)$, we have a set of seven equations in four independent variables, which can be solved when the data are available. The parameters $\epsilon_1$, $\epsilon_2$ and $\alpha$ can then be found, leaving one equation for the mixing parameters $a_i$ (table 2). Together with the normalization condition, this allows us to consider a mixture of two spin–orbital states. If $\alpha$ is found to be negligible, we can also put $\beta = 0$ in the equation for $\mu(\Lambda)$ and obtain a check on the various mixtures. Until the data are known, however, this cannot be taken any further. Three consistency relations are derivable:

$$\begin{aligned}\mu(\Sigma^+) - \mu(\Sigma^0) &= \mu(\Sigma^0) - \mu(\Sigma^-) \\ &= \mu(\mathrm{P}) + \tfrac{1}{2}\mu(\mathrm{N}) \\ \mu(\Xi^0) - \mu(\Xi^-) &= \mu(\mathrm{P}) + 2\mu(\mathrm{N}).\end{aligned}$$

On SU(3) symmetry there are, of course, six consistency relations (see, for example, Mayer 1963). In the present model only three of these would be satisfied accurately (to within a few per cent). This provides us with a test for our assumptions on SU(3) violation and the electromagnetic properties of quarks.

A case of special interest is obtained by restricting the wave function to the **56**, i.e. to the second and fifth columns of table 2. If spin–orbit forces predominate, this may be a reasonable description. From equations (11) we can then obtain $a_4$, together with five consistency relations: the results are given in the appendix. For a first approximation, we further assume zero mixing and put $a_1 = 1$. We can then solve for $\epsilon_1$ and $\epsilon_2$, using the nucleon magnetic moments, and obtain the six remaining moments as predictions:

$$\epsilon_1 = 380 \text{ MeV}, \qquad \epsilon_2 = 280 \text{ MeV}$$

$$\mu(\Sigma^+) = 2{\cdot}64 \text{ n.m.}, \qquad \mu(\Sigma^0) = 0{\cdot}80 \text{ n.m.}, \qquad \mu(\Sigma^-) = -1{\cdot}03 \text{ n.m.}$$
$$(2{\cdot}79 \text{ n.m.}) \qquad (0{\cdot}96 \text{ n.m.}) \qquad (-0{\cdot}88 \text{ n.m.})$$
$$\mu(\Lambda) = -0{\cdot}60 \text{ n.m.}, \qquad \mu(\Xi^0) = -1{\cdot}47 \text{ n.m.}, \qquad \mu(\Xi^-) = -0{\cdot}44 \text{ n.m.}$$
$$(-0{\cdot}96 \text{ n.m.}) \qquad (-1{\cdot}91 \text{ n.m.}) \qquad (-0{\cdot}88 \text{ n.m.}).$$

The values found on SU(3) symmetry are given for comparison. Estimates have been obtained previously (Bég and Pais 1965) by correction for the true baryonic mass according to

$$\mu' = \mu \frac{m\,(\text{nucleon})}{m\,(\text{hyperon})}.$$

Most of our estimates are significantly different from these; in particular, attention is drawn to $\mu(\Sigma^-)$ and $\mu(\Xi^-)$.

With the same assumptions, magnetic moments are readily calculated for vector mesons and decuplet baryons (K. R. James 1967, unpublished). As might be expected, non-zero moments are found for neutral particles of non-zero strangeness. In the approximation that these particles belong purely to S states, with vanishing vector potentials, we find

$$\mu(\Xi^{*0}) = 2\mu(Y_1^{*0}) = 0{\cdot}2 \text{ n.m.}$$
$$\mu(\bar{K}^{*0}) = -\mu(K^{*0}) = 0{\cdot}1 \text{ n.m.}$$

The present model allows us to calculate the magnetic moments of hadrons, including ordinary spin and SU(3) violation, according to simple and well-known methods. The model is based on strong assumptions, but consistency relations provide a test for its usefulness and distinguish it from other possible models. The situation is more favourable than for three-body nuclei, because more data are potentially available. If the method is valid, it may therefore lead to information on the spin-orbital structure of baryons. In a special case, restricted to the **56**, the results have been found explicitly.

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## Appendix

If the wave function is restricted to the **56**, the cross terms $\alpha$ and $\beta$ vanish (Sachs 1947), and we are left with three parameters: $\epsilon_1$, $\epsilon_2$ and $a_4$. Taking the independent moments to be $\mu$(P), $\mu$(N) and $\mu(\Lambda)$, and neglecting $a_4{}^2$, we obtain the following predictions from equations (11):

$$\mu(\Sigma^+) = A + 2{\cdot}45, \qquad \mu(\Sigma^0) = A + 0{\cdot}61, \qquad \mu(\Sigma^-) = A - 1{\cdot}22$$
$$\mu(\Xi^0) = -B - 0{\cdot}69, \qquad \mu(\Xi^-) = -B + 0{\cdot}34$$

where

$$A = \frac{0{\cdot}345\epsilon_a}{\epsilon_a + 980}, \qquad B = \frac{1{\cdot}22\,\epsilon_b}{\epsilon_b + 1520}$$

$$\epsilon_a = 1220(1-2a_4), \qquad \epsilon_b = 2730(1-a_4)$$

$$a_4 = 0{\cdot}47\,\frac{\mu(\Lambda)+0{\cdot}60}{\mu(\Lambda)+0{\cdot}88}.$$

If the amplitude is to be real, the last relation implies $|\mu(\Lambda)| \leqslant 0{\cdot}60$ n.m. This is within the present experimental limits (Hill *et al.* 1965, A. H. Rosenfeld *et al.* 1967, unpublished).

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